OPERATIONS RESEARCH (18MEC316): 2 MARKS QUESTIONS

UNIT -1: LINEAR PROGRAMMING MODELS

1. Define Linear programming. May 2016

Linear Programing is a technique for the optimization of a linear objective function subjected to linear equality or inequality constraints.

2. Define feasible solution and optimum solution.
    - The solution which satisfies all the constraints is known as feasible solution.
    - The best feasible solution is called optimal solution.
3. **Slack variable:** it is the variable added to less than or equal to type constraint equation to convert it in to equality type equation.
4. **Surplus variable:** it is the variable used/added to greater than or equal to type constraint equation to convert it in to equality type equation.
5. **Interpretation of dual variable:** it represents the unit worth of a resource.

6. Write the standard mathematical formulation to LPP. (May 2018)

General form of linear programming problem:

If objective function is maximization

$$Max\ \ Z = c_1x_1 + c_2x_2 + \ldots\ldots\ldots\ldots + c_nx_n$$

$$subjected\ \ to$$

$$\begin{aligned}
&a_{11}x_1 + a_{12}x_2 + \ldots\ldots\ldots + a_{1n}x_n \le b_1 \\
&a_{21}x_1 + a_{22}x_2 + \ldots\ldots\ldots + a_{2n}x_n \le b_2 \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
&a_{m1}x_1 + a_{m2}x_2 + \ldots\ldots\ldots + a_{mn}x_n \le b_m \\
&x_1, x_2, \ldots\ldots\ldots\ldots\ldots\ldots, x_n \ge 0
\end{aligned}$$

If objective function is minimization

$$Min\ \ Z = c_1x_1 + c_2x_2 + \ldots\ldots\ldots\ldots + c_nx_n$$

$$subjected\ \ to$$

$$\begin{aligned}
&a_{11}x_1 + a_{12}x_2 + \ldots\ldots\ldots + a_{1n}x_n \ge b_1 \\
&a_{21}x_1 + a_{22}x_2 + \ldots\ldots\ldots + a_{2n}x_n \ge b_2 \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
&a_{m1}x_1 + a_{m2}x_2 + \ldots\ldots\ldots + a_{mn}x_n \ge b_m \\
&x_1, x_2, \ldots\ldots\ldots\ldots\ldots\ldots, x_n \ge 0
\end{aligned}$$

7. what are the characteristics or properties a liner programming problem should have? May 2017

All linear programming problems must have following five characteristics:

(a) Objective function:

There must be clearly defined objec•tive which can be stated in quantitative way. In business problems the objective is generally profit maximization or cost minimization.

(b) Constraints:

All constraints (limitations) regarding resources should be fully spelt out in mathematical form.

(c) Non-negativity:

The value of variables must be zero or positive and not negative. For example, in the case of production, the manager can decide about any particular product number in positive or minimum zero, not the negative.

(d) Linearity:

The relationships between variables must be linear. Linear means proportional relationship between two ‘or more variable, i.e., the degree of variables should be maximum one.

(e) Finiteness:

The number of inputs and outputs need to be finite. In the case of infinite factors, to compute feasible solution is not possible.

8. What are the assumptions in Linear programming problem? May 2016

Assumptions Linear Programing Problem (LPP): **A linear programing problem (LPP) or Linear Programming Model (LP model) is based the assumptions of proportionality, additivity, continuity, certainty and finite choices.**

Proportionality:

A basic assumption in LP model is that proportionality exists in the objective function and the constraint inequalities. [If number of products produced is increased then proportionally the profit (contribution increases in objective function also. Similarly, the time required to produce also increases according to increase in quantity in constraint equations].

Additivity:

Another assumption underlying in LP model is that in the objective function and the constraint inequalities both, the total of all the activities is given by the sum total of each activity conducted separately. This assumption implies that there is no interaction between the decision variables.

Continuity: The decision variables are continuous.

Certainty:

The objective function coefficients, the coefficients of the inequality/equality constraints and the constraint (resource) values are constant. i.e known with certainty.

Finite choice:

LP model assumes that a limited number of choices are available to a decision maker and the decision variables do not assume negative values.

9. List out Phases of OR. **Dec 2017**

Seven Steps or phases of OR .

An OR project can be split in the following seven steps:

Step 1: Formulate the problem

The OR analyst first defines the organization's problem. This includes specifying the organization's objectives and the parts of the organization (or system) that must be studied before the problem can be solved.

Step 2: Observe the system

Next, the OR analyst collects data to estimate the values of the parameters that affect the organization's problem. These estimates are used to develop (in Step 3) and to evaluate (in Step 4) a mathematical model of the organization's problem.

Step 3: Formulate a mathematical model of the problem

The OR analyst develops an idealized representation — i.e. a mathematical model — of the problem.

Step 4: Verify the model and use it for prediction

The OR analyst tries to determine if the mathematical model developed in Step 3 is an accurate representation of the reality. The verification typically includes observing the system to check if the parameters are correct. If the model does not represent the reality well enough then the OR analyst goes back either to Step 3 or Step 2.

Step 5: Select a suitable alternative

Given a model and a set of alternatives, the analyst now chooses the alternative that best meets the organization's objectives. Sometimes there are many best alternatives, in which case the OR analyst should present them all to the organization's decision-makers, or ask for more objectives or restrictions.

Step 6: Present the results and conclusions

The OR analyst presents the model and recommendations from Step 5 to the organization's decision-makers. At this point the OR analyst may find that the decision makers do not approve of the recommendations. This may result from incorrect definition of the organization's problems or decision-makers may disagree with the parameters or the mathematical model. The OR analyst goes back to Step 1, Step 2, or Step 3, depending on where the disagreement lies.

Step 7: Implement and evaluate recommendation

Finally, when the organization has accepted the study, the OR analyst helps in implementing the recommendations. The system must be constantly monitored and updated dynamically as the environment changes. This means going back to Step 1, Step 2, or Step 3, from time to time.

Unit – 2: TRANSPORTATION PROBLEM AND ASSIGNMENT PROBLEMS

1. Define balanced and unbalanced Transportation problem? May 2016
   If the sum of supply = sum of demand , then it is called a balanced Transportation problem. If supply is not equal to demand, then it is called unbalanced Transportation problem.
2. What is degeneracy in Transportation Problem? Dec 2017
   If the number of occupied cells in a transportation problem is less than $m+n-1$, then degeneracy occurs in that problem. Such a solution is called degenerate solution.
   - To overcome this, we add infinitesimally small quantity to one (or more, if the need be) the least cost independent empty cell and treat this cell as an occupied cell.
3. What do you mean by travelling salesman problem?
   A traveling salesman wishes to go to a certain number of destinations in order to sell objects. He wants to travel to each destination exactly once and return home taking the shortest total route.
4. Explain North West corner rule with example?
5. Explain least cost cell method with example. May 2017
6. Explain Vogels Approximation method. May 2016

7. Write mathematical model of transportation problem?
   The transportation model can also be portrayed in a tabular form by means of transportation table shown in table 3.1.

Table 3.1 Transportation Table

Origin(i)	Destination(j)				Supply(a_i)
	1	2	…	n	
1	x_{11} c_{11}	x_{12} c_{12}		x_{1n} c_{1n}	a_1
2	x_{21} c_{21}	x_{22} c_{22}		x_{2n} c_{2n}	a_2
…	…	…	…	…	…
M	x_{m1} c_{m1}	x_{m2} c_{m2}		x_{mn} c_{mn}	a_m
Demand(b_j)	b_1	b_2	…	b_n	$\sum a_i = \sum b_j$

Mathematical model:

Let a_i = quantity of product available at origin i

b_j = quantity of product required at destination j

c_{ij} = the cost of transporting one unit of product from origin i to destination j

x_{ij} = the quantity transported from origin i to destination j

Minimize (Total cost) $Z = c_{11}x_{11} + c_{12}x_{12} + \ldots\ldots\ldots\ldots + c_{1n}x_{1n} +$

$c_{21}x_{21} + c_{22}x_{22} + \ldots\ldots\ldots\ldots + c_{2n}x_{2n} + \ldots\ldots\ldots + c_{m1}x_{m1} + c_{m2}x_{m2} + \ldots\ldots\ldots\ldots + c_{mn}x_{mn}$

subjected to

$$x_{11} + x_{12} + \ldots\ldots\ldots\ldots + x_{1n} = a_1$$
$$x_{21} + x_{22} + \ldots\ldots\ldots\ldots + x_{2n} = a_2$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$x_{m1} + x_{m2} + \ldots\ldots\ldots\ldots + x_{mn} = a_m$$
$$x_{11} + x_{21} + \ldots\ldots\ldots\ldots + x_{m1} = b_1$$
$$x_{12} + x_{22} + \ldots\ldots\ldots\ldots + x_{m2} = b_2$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$x_{1n} + x_{2n} + \ldots\ldots\ldots\ldots + x_{mn} = b_n$$
$$x_1,\ x_2,\ \ldots\ldots\ldots\ldots\ldots\ldots,\ x_n \geq 0$$

(Or)

$$Minimize\text{, Totalcost, } Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} x_{ij}$$

subject to

$$\sum_{j=1}^{n} x_{ij} = a_i \qquad for \ \ i = 1,2,\ldots, m$$

$$\sum_{i=1}^{m} x_{ij} = b_j \qquad for \ \ j = 1,2,\ldots, n$$

$x_{ij} \geq 0 \quad for\,all \ \ i = 1,2,\ldots,m \ \ and \ \ j = 1,2,\ldots,n$

8. Write mathematical model of Assignment problem.

Minimize (Total cost) $Z = c_{11}x_{11} + c_{12}x_{12} + \ldots\ldots\ldots\ldots + c_{1n}x_{1n} +$

$c_{21}x_{21} + c_{22}x_{22} + \ldots\ldots\ldots\ldots + c_{2n}x_{2n} + \ldots\ldots\ldots + c_{m1}x_{m1} + c_{m2}x_{m2} + \ldots\ldots\ldots\ldots + c_{mn}x_{mn}$

subjected to

$x_{11} + x_{12} + \ldots\ldots\ldots\ldots + x_{1n} = 1$

$x_{21} + x_{22} + \ldots\ldots\ldots\ldots + x_{2n} = 1$

....

....

$x_{m1} + x_{m2} + \ldots\ldots\ldots\ldots + x_{mn} = 1$

$x_{11} + x_{21} + \ldots\ldots\ldots\ldots + x_{m1} = 1$

$x_{12} + x_{22} + \ldots\ldots\ldots\ldots + x_{m2} = 1$

....

....

$x_{1n} + x_{2n} + \ldots\ldots\ldots\ldots + x_{mn} = 1$

$x_1, x_2, \ldots\ldots\ldots\ldots\ldots\ldots, x_n$ *are either zero or one*

(Or)

$$Minimize\text{, Totalcost, } Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} x_{ij}$$

subject to

$$\sum_{j=1}^{n} x_{ij} = 1 \qquad for \ \ i = 1,2,\ldots., m$$

$$\sum_{i=1}^{m} x_{ij} = 1 \qquad for \ \ j = 1,2,\ldots., n$$

$x_{ij} = 0 \ \ or 1\,; \quad for\,all \ \ i = 1,2,\ldots,m \ \ and \ \ j = 1,2,\ldots,n$

9. Explain Hungarian assignment method. Dec 2017, May 2017

   This method is applied for balanced assignment problem and is explained below.

   Step 1: locate the smallest cost element in each row of the cost table. Now subtract this smallest element from each element in that row. As a result, there shall be at least one zero in each row of this new table, called the reduced cost table.

   Step 2: in the reduced cost table obtained, consider each column and locate the smallest element in it. Subtract the smallest element from every other entry in the column.

   Step 3: draw the minimum number of horizontal and vertical lines (not the diagonal ones) that are required to cover all the zero elements. If the number of lines drawn is equal to "n" (number rows or columns) the solution is optimal, and proceed to step 6. If the number of lines drawn is smaller than "n" go to step 4.

   Step4: Select the smallest uncovered (by the lines) cost element. Subtract this element from all other elements including itself and add this element to each value located at the intersection of any two lines. The cost elements through which only one line passes remain unaltered.

   Step 5: Repeat steps 3 and 4 until an optimal solution is obtained.

   Step 6: given the optimal solution, make the job assignments as indicated by the zero elements. This is done as follows:

   a. Locate a row which contains only one zero element. Assign the job corresponding to this element to its corresponding person. Cross out the zeros, if any, in the column corresponding to the element.

   b. Repeat (a) for each of such rows which contain only one zero. Similarly, perform the same operation in respect of each column containing only one zero element, crossing out the zero(s), if any, in the row in which the element lies.

   c. If there is no row or column with only a single zero element left, then select a row or column arbitrarily and choose one of the jobs (or persons) and make the assignment. Now cross the remaining zeroes in the column and row in respect of which the assignment is made.
   d. Repeat steps (a) through (c) until all assignments are made
   e. Determine the total cost with reference to the original cost table.

Unit – 3: Network Models and SEQUENCING MODELS.

1. Define minimum spanning tree problem.
    - A tree is a subgraph that contains " n" vertices and "n-1" arcs.
    - A minimum spanning tree is a tree that covers all the vertices in the graph with minimum weight.
2. What is job shop sequencing and flow shop sequencing? (May 2016)

    Flow shop sequencing: in this all jobs follows same sequence of operations.
3. Distinguish CPM and PERT. (May 2016)

Sl. No.	CPM	PERT
1	CPM means Critical Path Method	PERT means Programme Evaluation and Review Technique
2	It is deterministic model	PERT is Probabilistic model
3	CPM is activity oriented approach	It is event oriented approach
4	It is useful for projects which are repetitive and standardized like construction activities	It is useful for projects which are new and non-repetitive like Research and Development projects.

4. What are the three time estimates used in PERT Network? Define them. (Dec 2017, May 2017)

    The three time estimates used in PERT network are Optimistic time, Pessimistic time and most likely time.

    Optimistic time (t_o): this is the shortest time the activity can take to complete.

    Most likely time (t_m): this refers to the time that would be expected to occur most often. complete.

    Pessimistic time (t_p): this is the longest time the activity can take to complete.
    - All the above time estimates follows Beta-distribution (Important for GATE)
5. List out network models:
    - Shortest route / path problem,
    - Minimum spanning tree ,
    - Maximum flow models,
    - transportation model and
    - Assignment model.

    Job shop sequencing: in this all jobs does not follow the same sequence of operations.
6. Objective of sequencing models is to find optimum processing order of jobs so that the total time required to complete all Jobs (makespan) and idle times of machines is minimum
7. Write the assumptions in sequencing models..
    a. Only one operation is carried out on a machine at a particular time
    b. Each operation once started must be completed.
    c. Only one machine of each type is available

d. A job is processed as soon as possible, but only in the order specified
e. An operation must be completed before its succeeding operation can start
f. Processing times are independent of order of performing the operation
g. The transportation time is negligible i.e. the time required to transport jobs from one machine to another is negligible
h. Jobs are completely known and are ready for processing when period under consideration starts.

8. When do you use dummy activity?
   It is used to satisfy the immediate predecessors and successors relationships among the activities in the network.
9. What is critical path? Critical activities?
   The longest path in the network diagram is called critical path. The activities on critical path are called critical activities. If you change the duration of these activities or delay these activities, it affect the project completion time. Hence, they are called critical activities. Float or slack for these critical activities is zero.
10. Applications of CPM / PERT
    These methods have been applied to a wide variety of problems in industries and have found acceptance even in government organizations. These include
    - Construction of a dam or a canal system in a region
    - Construction of a building or highway
    - Maintenance or overhaul of airplanes or oil refinery
    - Space flight
    - Cost control of a project using PERT / COST
    - Designing a prototype of a machine
    - Development of supersonic planes
11. List the Rules for constructing network diagrams. Dec 2016.
    - Each defined activity is represented by only one arrow in the network. Therefore, no single acitivity can be represented more than once in the network.
    - Before an activity can be undertaken all activities preceeding it must be completed.
    - The arrow direction indicates the general progression in time.
    - A network should have only one initial and one terminal node.
    - Try to avoid arrows which cross each other
    - Use straight arrows
    - Do not attempt to represent duration of activity by its arrow length

- Use arrows from left to right. Avoid mixing two directions, vertical and standing arrows may be used if necessary.
- Use dummies freely in rough draft but final network should not have any redundant dummies.

12. Define float or total float. Explain different types of floats.

    Total float of an activity represents the amount of time by which it can be delayed without delaying the project completion date.

    Total float = Latest Finish Time (LFT) – Earliest Finish Time (EFT)

    (or)

    = Latest Start Time (LST) – Earliest Start Time (EST)

    Types of floats:

    Interference float:

    Utilization of the float of an activity may affect the float times of the other activities. The part of the total float which causes a reduction in the float of the successor activities is called Interference float.

    Free Float:

    The part of the total float which can be used without affecting the float of the succeeding activities is called free float.

    Independent float:

    The independent float time of an activity is the amount of float time which can be used without affecting either the head or the tail events.

    It represents the amount of float time available for an activity when its preceding activities are completed at their latest and its succeeding activities begin at their earliest time.

    Independent float = EST for following activity – LST of preceding activity – duration of present activity

UNIT – 4; Game Theory

1. What is a saddle point? (Dec 2016, May 2017, May 2018)

   It is an equilibrium point at which minimax value is equal to maximin value.

   Max(Min) = Min(Max)
2. Define the Value of the game? (May 2017)

   The final pay-off to the winning player by the losing player is called the value of the game.
3. What is a fair game?

   If the value of the game is equal to zero, then it is called fair game.
4. What is strategy? (May 2018)

   A strategy refers to the action to be taken by a player in various contingencies (circumstances or possible situations) in playing the game.
5. What is dominance rule or dominance property in game theory? (Dec 2017, May 2016, Dec

2016)

In game theory, one strategy for a player is better than another strategy. We consider better strategy and delete the other one is called dominance rule.

6. What is two persons zero-sum game? (May 2018)

   In this game two players involved and gain of one player is equal to loss of another player. The sum of gain of a player and loss of another player is equal to zero.
7. What is non-zero sum game?

   In this game gain of one player is not equal to loss of another player.

   The sum of gain of a player and loss of another player is not equal to zero.
8. What is the optimum strategy for a player?

   It is the strategy at which the player gets maximum pay-off.
9. Define pay-off. (Dec 2016)

   Pay-off is the outcome of possible choice of strategies made by the players.
10. Explain maximin and minimax principle or strategy? (Dec 2017, May 2017)
    - **Minimax** (sometimes called **MinMax**) is a decision rule used in game theory, for *mini*mizing the possible loss or *max*imum loss)
    - "maximin" strategy used to maximize the minimum gain.
11. What are the strategies used to reduce the size of a game?

    Dominance principle and graphical methods are used to reduce the game.
12. What are the Assumptions in game theory? (Dec 2016)
    - It is assumed that players within the game are rational and will strive to maximize their payoffs in the game.
    - The players in the game knows the pay-off matrix
    - The number of players (competitors) is finite.
    - All players act rationally and intelligently.
    - Each player has a definite course of action.
    - There is conflict of interest between the players.
    - The rules of play are known to all the players.
13. What is game theory?

    Game theory is a body of knowledge which is concerned with the study of decision making in situations where two or more rational opponents are involved under conditions of competition and conflicting interests.
    - **A game refers a situation in which two or more players are competing.**
14. What are the various applications of game theory ? Dec 2016

    The game theory can be applied to decide the best course in conflicting situations. In business decisions it has wider possibi•lities. With the help of computer large number of independent variables can be considered with mathematical accuracy. The main advantages of this theory are:
    - Game theory provides a systematic quantitative approach for deciding the best strategy in competitive situations.
    - It provides a framework for competitor's reactions to the firm actions.

- It is helpful in handling the situation of independence of firms.

- Game theory is a management device which helps rational decision-making.

15. What are the various limitation of game theory?

    Limitations:

    - As the number of players increases in the actual business the game theory becomes more difficult.
    - It simply provides a general rule of logic not the winning strategy.
    - There is much uncertainty in actual field of business which cannot be considered in game theory.
    - Businessmen do not have adequate knowledge for the game theory.

UNIT – 5: QUEING THEORY

1. Explain the terms: Bulk Arrival, Balking, Reneging, Jockeying and patient customer. (May 2016, May 2017)

   Bulk Arrival: the arrival of customers in groups for service is called Bulk arrival or batch arrivals. Examples: families visiting restaurants, ship discharging cargo at dock.

   Balking: some customers arrive to the system for service but they will not join the queue for some reason and decide to come at a later time for service. This is known as balking.

   Reneging: some customers arrive to the system for service and they join the queue for service but they leave the system without taking service. This is known as Reneging.

   Jockeying: when customers come to multiple servers with multiple queues, they looks for shortest queues and joins for service. Sometimes customers move from one queue to other queues which are moving fast. This is known as Jockeying.

   Patient Customer: Customer who wait in the system still he gets the service is called Patient customer.

2. Define service utilization factor or traffic intensity or clearing ratio in queuing systems.(Dec 2016, Dec 2017)

   $let\ \lambda = arrival\ rate$

   $\mu = service\ rate,\ then$

   $service\ utilization\ factor,\ \rho = \dfrac{\lambda}{\mu}$ for single server models

   $\rho = \dfrac{\lambda}{C\mu}$ for multiple server models

   where $C =$ number of servers

3. List out Various notations used in the Queuing system or characteristics of a queuing system. May 2017

   n = Number of customers in the system
   p_n = Probability of exactly ' n' customers in the system
   W_q - Average time a customer spends waiting in the queue
   W_s - Average time a customer spends in the system
   L_q - Average number of customers in the queue
   L_s - Average number of customers in the system
   ρ = Utilisation factor for the service system
   μ =Mean number of customers served per time period
   λ = Mean number of arrivals per time period

4. Features of a queuing system or elements in queuing system. Dec 2016

   The essential features of queuing systems are:

   1) Calling population – it may be finite or infinite

   2) Arrival process – describes the arrival of customers to the system. In Queuing theory, it is assumed that customer arrivals follows poison distribution

   3) queue configuration,

   4) Queue discipline: this represents the order in which the customers are picked up from the waiting line for service. The following are the possibilities.

   - FCFS – First Come First Served
   - LCFS – Last Come First Served
   - SIRO – Service In Random Orders
   - Priority service

   5) Service process - describes the service given to the customers, Structure and speed of the service. In Queuing theory, it is assumed that service process follows exponential distribution.

5. Write the Kendal`s Notation of a Queuing models. Dec 2017

   Kendals notationsof a Queuing model is as follows
   A/B/C/D/E
   where A represents the distribution of the customer arrivals
   B = the distribution of service process
   C = number of servers in the systems
   D = Queue length (finite or infinite)
   E = Population (finite or Infinite)

QUEUEING THEORY

Characteristics of a queueing system:

A queueing system can be completely described by the following characteristics.

(i) Input Process:

This gives the mode of arrival of the customers into the system. Generally the customers arrive in a random manner. Hence the distribution of inter-arrival time follows some probability law. We assume that the customers ~~to~~ arrives in a system follows Poisson distribution.

(ii) Queue discipline: This is the law according to which the customers are served. The following are the different rules.

(a) FIFO (Fist In First Out):

The customers are served according to their arrival to the system.

examples: ration shops, ticket booking etc.

(b) LIFO (Last In First Out):

according to this rule, the unit or customer which arrives last is served first.

example: Passenger who gets into a crowded bus as the last man gets down first.

(c) SIRO (Service In Random Order).

In this rule customers are selected for service at random irrespective of their arrival time.

(iii) Service mechanism: This is the facility available in the system to serve the customers.

In certain cases one server may find it difficult when there is a large number of customers in a shop.

Then the number of Servers may be increased in order to reduce the waiting time of the customers.

(iv) Capacity of the System:

This is the capacity of the queue or waiting line. It represents the number of customers can be there in the queue.

Generally a queue is of infinite capacity. In some systems the customers are admitted into a waiting room whose capacity is limited.

Kendall's notion of a queueing model.

A | B | C | D | E

A represents arrival process

B represents service process

C represents number of Servers

D represents queued length

E represents Population.

M/M/1/∞/∞ model. (Single Server Infinite queuelength model).

Symbols:

n — number of Customers in the system both waiting and in service

λ = average number of Customers arriving per unit time (arrival rate)

μ = average number of customers served per unit time (service rate)

$\rho = \frac{\lambda}{\mu}$ (traffic density)

C = number of Servers. = 1

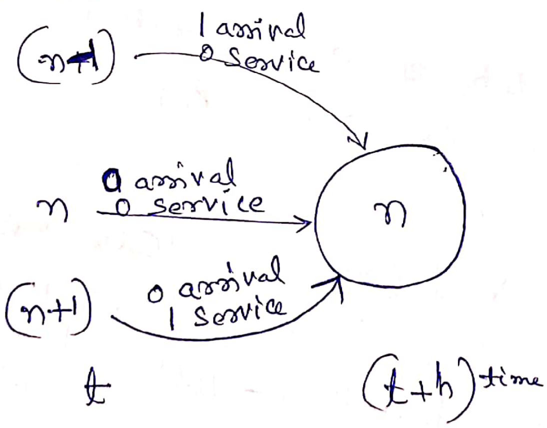
L_s = Average number of customers in the system.

L_q = Average number of Customers in the queue

$P_n(t)$ = Probability of n customers in the system at time t.

Assumption :- During small interval of time (h) only one event occurs.

Formula for steady state Probabilities:



$$P_n(t+h) = P_n(t) * P(\text{no arrival in } h) * P(\text{no service in } h)$$
$$+ P_{(n+1)}(t) * P(\text{no arrival in } h) * P(\text{one service in } h)$$
$$+ P_{(n-1)}(t) * P(\text{one arrival in } h) * P(\text{no service in } h)$$

Note: Probability of one arrival in $h = \lambda h$

P (one arrival in h) $= \lambda h$

P (no arrival in h) $= 1 - \lambda h$

P (one service in h) $= \mu h$

P (no service in h) $= 1 - \mu h$

$$P_n(t+h) = P_n(t)(1-\lambda h)(1-\mu h) + P_{n+1}(t)\,\mu h(1-\lambda h) + P_{n-1}(t)\,\lambda h(1-\mu h)$$

$$= P_n(t)\left[1 - \mu h - \lambda h + \mu\lambda h^2\right] + P_{n+1}(t)\left[\mu h - \mu\lambda h^2\right] + P_{n-1}(t)\left[\lambda h - \mu\lambda h^2\right]$$

By neglecting higher order terms.

$$P_n(t+h) = P_n(t)\left[1 - \mu h - \lambda h\right] + P_{n+1}(t)(\mu h) + P_{n-1}(t)(\lambda h)$$

$$= P_n(t) - P_n(t)(\mu+\lambda)h + P_{n+1}(t)\,\mu h + P_{n-1}(t)\,\lambda h$$

$$P_n(t+h) - P_n(t) = h\left[P_{n-1}(t)\lambda + P_{n+1}(t)\mu - P_n(t)(\mu+\lambda)\right]$$

$$\frac{P_n(t+h) - P_n(t)}{h} = P_{n-1}(t)\lambda + P_{n+1}(t)\cdot\mu - P_n(t)(\mu+\lambda)$$

under steady state ~~probabilities~~ conditions.

$$0 = P_{n-1}(t)\lambda + P_{n+1}(t)\mu - P_n(t)(\mu+\lambda)$$

$$\lambda P_{n-1} + \mu P_{n+1} = (\lambda+\mu) P_n \quad \text{---} \quad (1)$$

Let $P_0(t+h)$ = Probability of 0 customers in the system in $(t+h)$ time

$P_0(t+h) = P_1(t) * P$(one service) P(no arrival) $+ P_0(t) * P$ (no arrival)

$$P_0(t+h) = P_1(t)(\mu h)(1-\lambda h) + P_0(t)(1-\lambda h)$$

$$= P_1(t)\,\mu h + P_0(t) - P_0(t)\lambda h \quad \text{[By neglecting higher order terms]}$$

$$\frac{P_0(t+h) - P_0(t)}{h} = P_1(t)\mu - P_0(t)\lambda$$

$$0 = P_1(t)\mu - P_0(t)\lambda \quad \text{[under steady state condition]}$$

$$\mu P_1 = \lambda P_0 \quad \text{---} \quad (2)$$

$$\mu P_1 = \lambda P_0 \quad \text{—— (2)}$$

$$P_1 = \frac{\lambda}{\mu} P_0$$

$$P_1 = \rho P_0$$

Put $n=1$ in equation (1)

$$\lambda P_0 + \mu P_2 = (\lambda+\mu) P_1$$

$$= \lambda P_1 + \mu P_1$$

$$\lambda P_0 + \mu P_2 = \lambda P_1 + \lambda P_0 \quad \left[\begin{matrix}\text{from (2)} \\ \mu P_1 = \lambda P_0\end{matrix}\right]$$

$$\mu P_2 = \lambda P_1$$

$$P_2 = \frac{\lambda}{\mu} P_1 = \rho P_1$$

$$= \rho P_1$$

$$= \rho \, \rho P_0$$

$$= \rho^2 P_0$$

$$P_1 = \frac{\lambda}{\mu} P_0 = \rho P_0$$

$$P_2 = \frac{\lambda}{\mu} P_1 = \rho P_1 = \rho^2 P_0$$

$$P_3 = \rho P_2 = \rho^3 P_0$$

$$\vdots$$

$$P_n = \rho^n P_0$$

Sum of the steady state Probabilities $= 1$

$$P_0 + P_1 + P_2 + \cdots \infty = 1$$

$$P_0 + \rho P_0 + \rho^2 P_0 + \rho^3 P_0 + \cdots \infty = 1$$

$$P_0 (1 + \rho + \rho^2 + \cdots \infty) = 1 \qquad \left[S_\infty = \left(\frac{a}{1-r}\right) \right.$$

$$P_0 \left(\frac{1}{1-\rho} \right) = 1$$

$$\left[\lambda < \mu \text{ or } \rho < 1 \right]$$

P_0 is valid under $\rho < 1$ or $\lambda < \mu$.

$$P_0 = 1 - \rho$$

$$P_n = \rho^n P_0 = \rho^n (1 - \rho)$$

Average number of customers in the system (L_s)
or Expected no. of customers in the system.

$$L_s = \sum_{n=0}^{\infty} n P_n = \sum_{n=0}^{\infty} n \cdot \rho^n P_0$$

$$= \sum_{n=0}^{\infty} n \cdot \rho^n (1-\rho)$$

$$= (1-\rho) \sum_{n=0}^{\infty} n \cdot \rho^n$$

$$= (1-\rho) \sum_{n=0}^{\infty} n \cdot \rho^{n-1} \cdot \rho$$

$$= \rho(1-\rho) \cdot \sum_{n=0}^{\infty} \cdot n\rho^{n-1}$$

$$= \rho(1-\rho) \cdot \sum \frac{d}{d\rho} \rho^n$$

$$= \rho(1-\rho) \cdot \frac{d}{d\rho} \sum \rho^n$$

$$= \rho(1-\rho) \cdot \frac{d}{d\rho} (1+\rho+\rho^2+\rho^3+ \cdots \infty)$$

$$= \rho(1-\rho) \cdot \frac{d}{d\rho} \left(\frac{1}{1-\rho}\right)$$

M/M/1/∞/∞ Model

$$= \rho(1-\rho) \cdot \frac{1}{(1-\rho)^2} = \frac{\rho}{1-\rho}$$

$$L_s = \frac{\rho}{1-\rho} \quad \text{--- (1)}$$

$\rho = \lambda/\mu$; $P_0 = 1-\rho$

$L_s = L_q +$ expected number of customers being served

$$= L_q + \frac{\lambda}{\mu} \quad \text{--- (2)}$$

$$L_s = \lambda W_s \quad \text{--- (3)}$$

$$L_q = \lambda W_q \quad \text{--- (4)}.$$

Note: Probability that the number of customers in the system is $\geq n$ $= \rho^n$

$$P(N \geq n) = \rho^n$$

209, Iyer A. T.V repair man finds that the time spent on repairing has an exponential distribution with mean 30 min per unit. The arrival of T.V sets is Poisson with an average of 10 sets per day of 8 hours. What is the expected Idle time per day? How many sets are there on the average?

Sol: given data: service time per unit = 30 min

μ, service rate = 2 per hr.

λ, arrival rate = $\frac{10}{8}$ per hour

$= \frac{5}{4}$ per hour

Probability that there is no unit in the system P_0

$P_0 = 1 - e$

$= 1 - \frac{\lambda}{\mu} = 1 - \frac{5/4}{2} = 1 - \frac{5}{8}$

$= \frac{3}{8}$

Idle time per day of 8 hours $= \frac{3}{8} \times 8$

$= 3$ hours.

Average number of units in the system, $L_s = \frac{e}{1-e}$

$e = \frac{\lambda}{\mu} = \left(\frac{5}{4}\right)/2 = \frac{5}{8}$

$L_s = \frac{5/8}{1-5/8} = \frac{5/8}{3/8} = \frac{5}{3}$ sets.

Note: Average number of sets in queue, $L_q = ?$

$L_s = L_q +$ sets being serviced

$= L_q + \frac{\lambda}{\mu}$

$L_q = L_s - \frac{\lambda}{\mu} = \frac{5}{3} - \frac{5/4}{2} = \frac{5}{3} - \frac{5}{8}$

$= \frac{40-15}{24} = \frac{25}{24}$ sets.

Set waiting time in the system $W_s = ?$

$L_s = \lambda W_s$

$W_s = \frac{L_s}{\lambda} = \frac{5}{3} / \frac{5}{4} = \frac{\cancel{5}}{3} \times \frac{4}{\cancel{5}} = \frac{4}{3}$ hrs

Set waiting time in queue, $W_q = ?$

$L_q = \lambda W_q \Rightarrow W_q = \frac{L_q}{\lambda} = \left(\frac{25}{24}\right) / \frac{5}{4} = \frac{\cancel{25}^{5}}{\cancel{24}_{6}} \times \frac{\cancel{4}^{1}}{\cancel{5}} = \frac{5}{6}$ hrs.

$L_q = \lambda W_q \Rightarrow W_q = \frac{L_q}{\lambda}$

PB2
209, I yer

In a railway yard goods trains arrive at the rate of 30 trains per day. Assuming that the service time follows exponential distribution with an average of 36 minutes find

(i) the Probability that the number of trains in the yard exceeds 10

(ii) Average number of trains in the yard.

Sol:

given data:

λ, Arrival rate $= 30$ trains Per day

$= \frac{30}{24}$ Per hour

$= \frac{5}{4}$ Per hr

Service rate, $\mu = \frac{60}{36}$ Per hour

$= \frac{10}{6}$ per hr

$= \frac{5}{3}$ per hr

$$\rho = \frac{\lambda}{\mu} = \left(\frac{5}{4}\right) / \left(5/3\right) = \frac{5}{4} \times \frac{3}{5} = \frac{3}{4}$$

(i) Probability that number of trains exceeds 10

$$P(N \geq 11) = \rho^{11} = \left(\frac{3}{4}\right)^{11}$$

(ii) Average number of trains in the yard, L_s

$$L_s = \frac{\rho}{1-\rho} = \frac{3/4}{1-3/4} = \frac{3/4}{1/4} = 3$$

Note: Average no. of Trains in the queue L_q

$$L_s = L_q + \frac{\lambda}{\mu}$$

$$L_q = L_s - \frac{\lambda}{\mu} = 3 - 3/4 = \frac{9}{4} \text{ Trains}$$

waiting time in the system; $W_s = ?$

$$L_s = \lambda W_s \Rightarrow W_s = \frac{L_s}{\lambda} = \frac{3}{5/4}$$

$$= \frac{3}{1} \times \frac{4}{5} = \frac{12}{5} \text{ hr}$$

Train waiting time in the queue, $W_q = ?$

$$L_q = \lambda W_q \Rightarrow W_q = L_q/\lambda$$

$$= \left(\frac{9}{4}\right) / \left(5/4\right)$$

$$= \frac{9}{4} \times \frac{4}{5} = \frac{9}{5} \text{ hr}$$

209, Iyer

In a store with one server, 9 customers arrive on an average of 5 minutes. Service is done for 10 customers in 5 minutes. Find

(i) the average number of customers in the system

(ii) the average queue length

(iii) the average time a customer spends in the store

(iv) the average time a customer waits before being served.

Sol: given data.

λ, arrival rate $= \frac{9}{5}$ customers per min

Service rate, $\mu = \frac{10}{5}$ customers per min
$= 2$ customers per min.

(i) the Average no. of customers in the system, L_s

$$L_s = \frac{\rho}{1-\rho}$$

$$\rho = \frac{\lambda}{\mu} = \frac{9/5}{2} = \frac{9}{5} \times \frac{1}{2} = \frac{9}{10}$$

$$L_s = \frac{9/10}{1-\frac{9}{10}} = \frac{\frac{9}{10}}{1/10} = 9 \text{ customers.}$$

(ii) the average queue length, L_q

$$L_s = L_q + \frac{\lambda}{\mu} \Rightarrow L_q = L_s - \frac{\lambda}{\mu}$$
$$= 9 - \left(\frac{9}{5}\right)/2$$
$$= 9 - \frac{9}{10} = \frac{81}{10} = 8.1 \text{ customers}$$

(iii) the average time a customer spends in the store W_s

$$L_s = \lambda W_s \Rightarrow W_s = \frac{L_s}{\lambda} = \frac{9}{9/5} \text{ min}$$
$$= 9 \times \frac{5}{9} = 5 \text{ min}$$

(iv). the average time a customer waits before being served. W_q

$$L_q = W_q \cdot \lambda$$

$$W_q = \frac{L_q}{\lambda} = \frac{8.1}{9/5} = 8.1 \times \frac{5}{9} = 4.5 \text{ min.}$$

PB 7

210, I yer

In a Carwash station cars arrive for service according to Poisson distribution, with mean 4 per hour. The average service time of a car is 10 min.

(i) Determine the probability that an arriving car has to wait

(ii) Find the average time a car stays in the station

(iii) If the parking space can not hold more than 6 cars, find the probability that an arriving car has to wait out side.

Sol:

arrival rate, $\lambda = 4$ per hr

Service time $= 10$ min

Service rate $= 6$ per hr

$\rho = \frac{\lambda}{\mu} = \frac{4}{6} = 2/3$

(i) Probability that an arriving car has to wait $= 1 - P_0 = \cancel{2/3}$

$P_0 = 1 - \rho = 1 - 2/3 = \frac{1}{3}$

$1 - P_0 = 1 - 1/3 = 2/3$

(ii) Average time a car stays in the station W_s

$L_s = \lambda W_s$

$L_s = \frac{\rho}{1-\rho} = \frac{2/3}{1-2/3} = \left(\frac{2}{3}\right)/\tfrac{1}{3} = 2$

$W_s = \frac{L_s}{\lambda} = \frac{2}{4}$ hr $= \frac{1}{2}$ hr $= 30$ min

(iii) If the Parking space is full, then there are 6 cars waiting for service and one car in service. Therefore $n = 7$. The new arrival has to wait out side if $n \geq 8$.

The required Probability $= P(N \geq 8) = \rho^8$

$= \left(\frac{2}{3}\right)^8$

PB
214, Iyer

At a railway station, only one train is handled at a time. The yard can accomidate only two trains to to wait. Arrival rate of trains is 6 per hour and the railway station can handle them at the rate of 12 per hour. Find the steady state probabilities for the various number of trains in the system. Also find the average waiting time of a newly arriving train.

Sol:

λ = arrival rate = 6 per hour

μ = Service rate = 12 per hour

Maximum queue length is 2 and the maximum number of trains in the system is $N = 3$.

So steady state probabilities upto P_3 we need to calculate.

$$P_0 = \frac{1-\rho}{1-\rho^{N+1}} \quad ; \quad \rho = \frac{\lambda}{\mu} = \frac{6}{12} = \frac{1}{2} = 0.5$$

$$= \frac{1-0.5}{1-(0.5)^{3+1}} = 0.53$$

$$P_1 = \rho P_0 = 0.5 \times 0.53 = 0.27$$

$$P_2 = \rho P_1 = \rho^2 P_0 = 0.5^2 \times 0.53 = 0.13$$

$$P_3 = \rho P_2 = \rho^3 P_0 = 0.5^3 \times 0.53 = 0.07$$

These are the probabilities for the various number of trains in the system.

Average number of trains in the system $= 0P_0 + 1P_1 + 2P_2 + 3P_3$

$= 0 + 1(0.27) + 2(0.13) + 3 \times 0.07 = 0.74$

Each train takes $1/12$ hour for service

Service time for one train $= \frac{1}{12}$ hr $= 0.085$ hr

A newly arriving train finds 0.74 trains in the system with service time $= 0.085$ hr each.

Expected waiting time $= 0.74 \times 0.085 = 0.0629$ hr

$= 3$ minutes.

MODEL : 2.

$M|M|1|N|\infty$ model (single server finite queue length model).

$P_0 + P_1 + P_2 + \cdots + P_N = 1$

$P_0 + \rho P_0 + \rho^2 P_0 + \cdots + \rho^N P_0 = 1 \Rightarrow P_0(1+\rho+\rho^2+\cdots+\rho^N) = 1$

$P_0\left[\frac{1(1-\rho^{N+1})}{1-\rho}\right] = 1$ ~~$a(1-r)$~~

(i) $$P_0 = \frac{1-\rho}{1-\rho^{N+1}} \quad (\rho \neq 1) \quad (\rho < 1)$$

Geometric Progression :

$S_n = a + ar + ar^2 + \cdots + ar^{n-1}$

Sum of first n terms.

$$S_n = \frac{a(1-r^n)}{1-r} \quad \text{if } r<1$$

(ii) $$P_N = \rho^n P_0 \quad (n = 1, 2, \cdots N)$$

$$L_s = \frac{\rho}{1-\rho}\left[\frac{1+N\rho^{N+1}-(N+1)\rho^N}{(1-\rho^{N+1})}\right]$$

$$L_s = L_q + \frac{\lambda_{effective}}{\mu}$$

$$\lambda_{effective} = \lambda(1-P_N)$$

$$L_s = \lambda_{eff} W_s$$

$$L_q = \lambda_{eff} W_q$$

$$P_0 = \frac{\rho-1}{\rho^{N+1}-1} \quad \text{if } (\rho > 1)$$

PB, 5

210

In a telephone booth, the arrivals follow Poisson distribution with an average of 9 minutes between two consecutive arrivals. The duration of telephone call is exponential with an average of 3 min.

(i) Find the Probability that a person arriving at the booth has to wait

(ii) Find the average queue length

(iii) Find the fraction of the day, the phone will be in use

(iv) The Company will install a second booth if a customer has to wait for phone, for at least 4 minutes. If so, find the increase in the flow of arrivals in order that another booth will be installed.

Sol:

Inter arrival time = 9 min

arrival rate, $\lambda = \frac{60}{9}$ per hr. $= \frac{20}{3}$ per hr $= \frac{1}{9}$ per min

Service rate, $\mu = \frac{60}{3}$ Per hr $= 20$ Per hr $= \frac{1}{3}$ Per min

$$\rho = \frac{\lambda}{\mu} = \frac{\frac{1}{9}}{\frac{1}{3}} = \frac{1}{9} \times \frac{3}{1} = \frac{1}{3}$$

(i) Probability that an arrival has to wait

$$= 1 - P_0$$
$$= 1 - (1 - \rho) = \rho$$
$$= \frac{1}{3}$$

(ii) Average queue length, L_q

$$L_s = L_q + \frac{\lambda}{\mu}$$

$$L_s = \frac{\rho}{1-\rho} = \frac{\frac{1}{3}}{1 - \frac{1}{3}} = \frac{1}{3} \times \frac{3}{2} = \frac{1}{2}$$

$$L_q = L_s - \frac{\lambda}{\mu} = \frac{1}{2} - \frac{1}{3} = \frac{3-2}{6} = \frac{1}{6}$$

(iii) The fraction of time the phone is busy

$$= 1 - P_0 = 1 - (1 - \rho) = \rho = \frac{1}{3}$$

(iv) Let λ_1 be the arrival rate for the average waiting time in the queue to be at least 4 min.

$$L_q = \lambda W_q \Rightarrow W_q = \frac{L_q}{\cdot}$$

$$W_q = \frac{L_q}{\lambda}$$

$$L_s = L_q + \lambda/\mu$$

$$L_q = L_s - \frac{\lambda}{\mu}$$

$$L_s = \frac{\rho}{1-\rho} = \frac{\lambda/\mu}{1-\frac{\lambda}{\mu}} = \frac{\frac{\lambda}{\mu}}{\frac{\mu-\lambda}{\mu}} = \frac{\lambda}{\mu-\lambda}$$

$$L_s = \frac{\lambda}{\mu-\lambda}$$

$$L_q = L_s - \frac{\lambda}{\mu} = \frac{\lambda}{\mu-\lambda} - \frac{\lambda}{\mu}$$

$$= \frac{\mu\lambda - \lambda(\mu-\lambda)}{\mu(\mu-\lambda)}$$

$$= \frac{\cancel{\mu\lambda} - \cancel{\lambda\mu} + \lambda^2}{\mu(\mu-\lambda)}$$

$$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)}$$

$$W_q = \frac{L_q}{\lambda} = \frac{\lambda^2}{\mu(\mu-\lambda)} \times \frac{1}{\lambda}$$

$$= \frac{\lambda}{\mu(\mu-\lambda)}$$

as per the given Problem

$$4 = \frac{\lambda_1}{\mu(\mu-\lambda_1)}$$

$$4 = \frac{\lambda_1}{\frac{1}{3}\left(\frac{1}{3}-\lambda_1\right)} =$$

$$\cancel{4} = \frac{\lambda_1}{\frac{1}{3}\left(\frac{1-3\lambda_1}{3}\right)}$$

$$= \frac{\lambda_1 \times 9}{(1-3\lambda_1)}$$

$$4 - 12\lambda_1 = 9\lambda_1$$

$$4 = 21\lambda_1 \Rightarrow \lambda_1 = 4/21$$

Second booth will be installed if arrival rate $= \frac{4}{21}$

Required increase in the flow of arrival

$$= \frac{4}{21} - \frac{1}{9} = \frac{12-7}{63} = \frac{5}{63} \text{ Per min.}$$

PB
214, I yer

The capacity of a queueing system is 4. Inter arrival time of the units is 20 min and the service time is 36 min per unit. Find the probability that a new arrival enters into service without waiting. Also find the average number of units in the system.

Sol:

Inter arrival time = 20 min

arrival rate $= \lambda = 3$ per hr

Service time $= 36$ min

service rate $= \mu = \frac{60}{36}$ per hr $= \frac{5}{3}$ per hr.

$N = 4$.

$\rho = \frac{\lambda}{\mu} = \frac{3}{5/3} = \frac{3}{1} \times \frac{3}{5} = \frac{9}{5} = 1.8$

(i) A new arrival can enter into service without waiting if the system is empty.

$$P_0 = \frac{1-\rho}{1-\rho^{N+1}} = \frac{\rho-1}{\rho^{N+1}-1} = \frac{1.8-1}{(1.8)^5-1} = 0.04$$

(ii) Average No. of units in the system:

$$0P_0 + 1P_1 + 2P_2 + 3P_3 + 4P_4$$

$$= 0.P_0 + 1P_1 + 2P_2 + 3P_3 + 4P_4$$

$$= 0 + \rho P_0 + 2\times\rho^2 P_0 + 3\cdot\rho^3 P_0 + 4\cdot\rho^4 P_0$$

$$= P_0(\rho + 2\rho^2 + 3\rho^3 + 4\rho^4)$$

$$= 0.04(1.8 + 2\cdot(1.8)^2 + 3(1.8)^3 + 4\cdot(1.8)^4)$$

$$= 2.71 \text{ (nearly 3)}$$

PB
214. I yer

Patients arrive at a clinic at the rate of 30 patients per hour. The waiting hall can not accommodate more than 14 patients. It takes 3 minutes on the average to examine a patient.

(i) Find the probability that an arriving patient need not wait.

(ii) Find the probability that an arriving patient finds a vacant seat in the hall.

Sol: Arrival rate $= \lambda = 30$ per hour

Service rate $= \mu = 20$ per hour

$$\rho = \frac{\lambda}{\mu} = \frac{30}{20} = 1.5 \quad (\rho > 1)$$

$$N = 14$$

(i) An arriving patient need not wait if the system is empty

$$P_0 = \frac{\rho - 1}{\rho^{N+1} - 1} = \frac{1.5 - 1}{(1.5)^{14+1} - 1} = 0.001$$

(ii) an arriving patient finds a vacant seat if the number of patients in the system is less that 14.

Hence the required Probability $= P_0 + P_1 + P_2 + \cdots + P_{13}$

$$= 1 - P_{14}$$

$$= 1 - \rho^{14} P_0 = 1 - (0.001)(1.5)^{14}$$

$$= 1 - 0.29 = 0.71$$

Scanned with Oken Scanner

PB

604, Kranti swaroop

A foreign bank is considering opening a drive-in window for customer service. Management estimates that customers will arrive for service at the rate of 12 per hr. The teller whom it is considering to staff the window can serve customers at the rate of one every three minutes. Assuming Poisson arrivals and Exponential service, find:

(i) utilization of teller (ii) Average number in the system

(iii) Average waiting time in the line

(iv) Average waiting time in the system.

Sol:

given data:

arrival rate, $\lambda = 12$ per hour

Service time = 3 min per customer

Service rate = 20 per hr.

(i) utilization of teller $\rho = \frac{\lambda}{\mu} = \frac{12}{20} = \frac{3}{5} = 0.6$

utilization of teller = 60%

(ii) Average number in the system, L_s

$$L_s = \frac{\rho}{1-\rho} = \frac{0.6}{1-0.6} = \frac{0.6}{0.4} = \frac{3}{2} \text{ customers.}$$

(iii) Average waiting time in the line: W_q

$$L_q = \lambda W_q$$

$$L_s = L_q + \frac{\lambda}{\mu}$$

$$\frac{3}{2} = L_q + \frac{3}{5}$$

$$L_q = \frac{3}{2} - \frac{3}{5} = \frac{15-6}{10} = \frac{9}{10}$$

$$W_q = \frac{L_q}{\lambda} = \frac{9/10}{12} = \frac{\cancel{9}^{3}}{10} \times \frac{1}{\cancel{12}_{4}} = \frac{3}{40} \text{ hr}$$

$$= \frac{3}{\cancel{40}_{4}} \times \cancel{60} \text{ min}$$

$$= \frac{9}{2} \text{ min}$$

(iv) Average waiting time in the system, W_s

$$L_s = \lambda W_s$$

$$W_s = \frac{L_s}{\lambda} = \frac{3/2}{12} \text{ hr} = \frac{\cancel{3}}{2} \times \frac{1}{\cancel{12}_{4}} \text{ hr}$$

$$= \frac{1}{8} \text{ hr} = \frac{1}{\cancel{8}_{\cancel{4}\,2}} \times \cancel{60}^{\cancel{30}\,15} \text{ min}$$

$$= 7.5 \text{ min}$$

PB 1. At a cycle repair shop on an average, a customer arrives every 5 minutes and on an average the service time is 4 min per customer. Suppose that the inter arrival time follows poisson distribution and the service times are exponentially distributed. Find L_s, L_q, W_s, W_q, shop busy and idle percentages. Assuming that there is only one server.

Sol: given data:

Inter arrival time = 5 min

arrival rate, $\lambda = 12$ per hr

average service time = 4 min

Service rate $\mu = 15$ per hr.

(i) % shop busy $= \rho = \frac{\lambda}{\mu} = \frac{12}{15} = 0.8$

% shop busy = 80%

% shop Idle $= 1 - 0.8 = 0.20$

$= 20\%$

(ii) Average Number of Customers in the System

$$L_s = \frac{\rho}{1-\rho} = \frac{0.8}{1-0.8} = \frac{0.8}{0.2} = 4 \text{ customers.}$$

(iii) Average Number of customers in the queue, L_q

$L_s = L_q + \lambda/\mu$

$4 = L_q + 0.8$

$L_q = 4 - 0.8 = 3.2$

(iv) Average waiting time of a customer in the System W_s =

$L_s = \lambda W_s \Rightarrow W_s = \frac{L_s}{\lambda} = \frac{4}{12}$ hr

$= \frac{4}{12} \times 60$ min

$= 20$ min

(v) Average waiting time of a customer in the line, W_q

$L_q = \lambda W_q \Rightarrow W_q = \frac{L_q}{\lambda} = \frac{3.2}{12}$ hr

$= 0.2667$ hr

$= 16$ min